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# The pion electromagnetic form-factor in a QCD-inspired model *

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**Abstract.** We present detailed numerical results for the pion space-like electromagnetic form factor obtained within a recently proposed model of the pion electromagnetic current in a confining light-front QCD-inspired model. The model incorporates the vector meson dominance mechanism at the quark level, where the dressed photon with $q^+ > 0$ decays in an interacting quark-antiquark pair, which absorbs the initial pion and produces the pion in the final state.

## 1 Introduction

The light-front dynamics (LFD) [1] is a suitable framework for describing electromagnetic (em) interactions and bound states for hadronic systems ([2]). Within light-front dynamics and in the framework of an inspired-QCD model [3], we have studied the pion em form factors in both space- and -time-like regimes [4]. In this contribution, detailed numerical results are presented and compared to the experimental data the electromagnetic form factor for the pion in the space-like region for $-q^2 \ < \ 10 \ (GeV/c)^2$.

In order to calculated the matrix elements of the pion em current we use the Mandelstam formula. In the space-like (SL) region one has

$$\begin{aligned} j^\mu \quad = \quad & -\mathrm{ie}\ 2\frac{m^2}{f_\pi^2} N_c \int \frac{d^4k}{(2\pi)^4} \overline{\Lambda}_{\pi'}(k+P_\pi, P_{\pi'}) \Lambda_\pi(k, -P_\pi) \\ & \times Tr\left[S(k+P_\pi)\gamma^5 S(k-q)\ \Gamma^\mu(k,q)\ S(k)\gamma^5\right] \ , \end{aligned} \tag{1}$$

where $S(k,p)$ is Dirac propagator for the quarks, $m$ is constituent quark mass, $N_c = 3$ is the number of colors and the factor 2 in the equation above, comes from

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isospin algebra. $\Gamma^\mu(k,q)$ is the quark-photon vertex, $q^\mu$ the momentum transfer and $\bar{\Lambda}_\pi$ the pion vertex function. The expression of the current appropriate for the time-like (TL) region can found in [4].

In principle, the complete vertex function for the photon and the pion should be obtained by solving an inhomogeneous and a homogeneous Bethe-Salpeter equation, respectively. In general, the pion vertex can contain a pseudovector spinor operator, besides the pseudoscalar one shown in Eq.(1) and adopted in what follows.

The relative light-front time in Eq. (1) is eliminated by performing the integration over $k^-$ $(k^0 - k^3)$ in a frame where $q^+ \neq 0$ $(q^+ = q^0 + q^3)$ and $\boldsymbol{q}_\perp = \boldsymbol{P}_{\pi'\perp} = \boldsymbol{P}_{\pi\perp} = \boldsymbol{0}_\perp$ [5]. We disregard the analytic structure of the vertex functions when performing the $k^-$ analytical integration. To further simplify the model calculations, the pion is considered massless. Due to the frame choice and the hypothesis of a massless pion, only the pair production mechanism survives in the photon absorption process. Mathematically the pair diagram comes from the residue of the integrand in Eq. (1) evaluated at $k^- = q^- - (q-k)^-_{on}$ (the on-minus-shell value) for $0 < k^+ < q^+$.

The hadronic part of the virtual-photon wave function is dominated by the vector-meson (VM) contribution, and this leads to a model for the quark-photon vertex given by a sum over the VM vertex function properly weighted. The momentum components of VM and pion vertex functions, in a constituent quark-antiquark approach, come from the front-form wave functions in a QCD-inspired model that describes the meson spectrum in both the light- and heavy-quark sectors [3].

## 2 Pion electromagnetic form factor

The electromagnetic form factor of the pion is the invariant defined from the general form of the matrix elements of the em current, as:

$$\begin{aligned} j^\mu_{SL} &= \langle \pi | \overline{q}\gamma^\mu q | \pi' \rangle = (P'^\mu_\pi + P^\mu_\pi) F_\pi(q^2) \ , \\ j^\mu_{TL} &= \langle \pi\overline{\pi} | \overline{q}\gamma^\mu q | 0 \rangle = (P'^\mu_\pi - P^\mu_\pi) F_\pi(q^2) \ , \end{aligned} \tag{2}$$

in the space-like and time-like regions, respectively.

The integration of the Mandelstam formula Eq. (1) requires the pion and the quark-photon vertex functions, which are evaluated within LFD. In the following, we present the main ingredients of our model calculation and refer to [4] for further details.

The quark photon vertex in Eq. (1) is constructed using the vector meson dominance (VMD) hypothesis. In the case of the plus component of the em current the quark-photon vertex is

$$\Gamma^+(k,q) = \sqrt{2} \sum_{n,\lambda} \left[ \epsilon_\lambda \cdot \widehat{V}_n(k, k-q) \right] \Lambda_n(k, P_n) \ \frac{[\epsilon^+_\lambda]^* f_{Vn}}{[q^2 - M_n^2 + \mathrm{i} M_n \Gamma_n(q^2)]} \ , \tag{3}$$

where $f_{Vn}$ is the decay constant of the n-th vector meson with mass $M_n$ and quadrimomentum $P_n$. $\Gamma_n(M_n^2)$ is the total width and $\epsilon_\lambda$ is the polarization of the

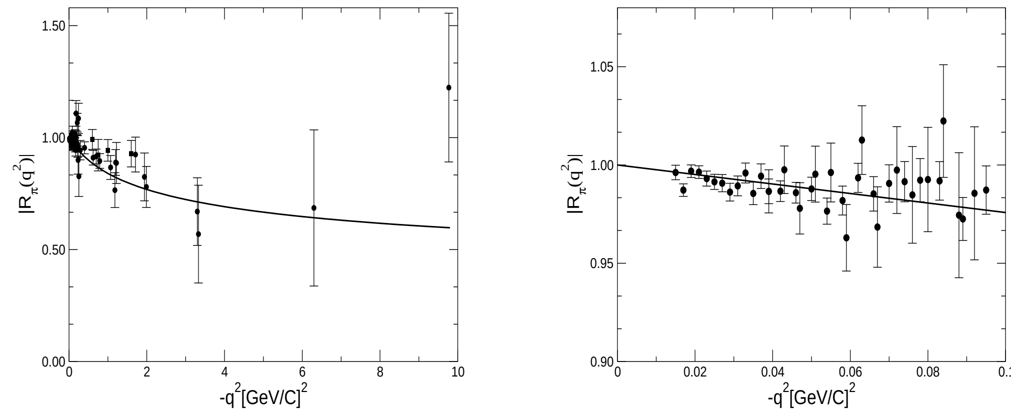


**Figure 1.** The ratio $R_\pi(q^2) = F_\pi(q^2)/(1 - q^2/m_\rho^2)$ vs $q^2$, in the SL region. Experimental data from Ref. [8].

vector meson. In Eq. (3), $\epsilon_\lambda \cdot \widehat{V}_n(k, k-q)\Lambda_n(k, P_n)$ is the VM vertex function, including the operator structure in the spinor space. We use the operator form $\widehat{V}_n^\mu(k, k-q) = \gamma^\mu - (k^\mu_{on} - (q-k)^\mu_{on})/(M_0 + 2m)$ [6], where for simplification all momentum are evaluated on-minus-shell and $M_0$ is the free mass of the virtual $q\bar{q}$ pair produced by the decay of the VM resonance.

Introducing Eq. (3) in Eq. (1), the electromagnetic form factor is written as

$$F_\pi(q^2) = \sum_n \frac{f_{Vn}}{q^2 - M_n^2 + \mathrm{i} M_n \Gamma_n(q^2)} \; g^+_{Vn}(q^2) \; , \tag{4}$$

where $g^+_{Vn}(q^2)$, for $q^2 > 0$, is the form factor for the VM decay in a pair of pions, and for $q^2 < 0$ is the form factor for the photon absorption through the coupling with a VM resonance.

The momentum components of the vector meson vertex $\Lambda_n(k, P_n)$ in Eq. (3) and the pion vertex $\overline{\Lambda}_{\pi'}(k + P_\pi, P_{\pi'})$ evaluated at $k^- = q^- - (q-k)^-_{on}$ in Eq. (1) are related to the valence component of the respective front-form wave function. The momentum dependence of the vertex for a hadron $h$ (a VM or a pion) is

$$\Lambda_h(k, P_h)]_{[k^- = q^- - (q-k)^-_{on}]} = \frac{C_h}{P_h^+} \left[ M_h^2 - M_0^2 \right] \psi_h(k^+, \mathbf{k}_\perp; P_h^+, \mathbf{P}_{h\perp}) \; , \tag{5}$$

where $\psi_h(k^+, \mathbf{k}_\perp; P_h^+, \mathbf{P}_{h\perp})$ is an eigenstate of a QCD-inspired mass operator [3]. The vertex $\Lambda_\pi(k, -P_\pi)$ evaluated at $k^- = q^- - (q-k)^-_{on}$ represents a three-body system, composed by a radiated pion plus a $q\bar{q}$ pair, as described in Ref. [7]. Such a vertex is given by a pseudoscalar coupling multiplied by a constant, fixed through the charge normalization.

The vector decay constant $f_{Vn}$ is calculated from the covariant expression $\epsilon^\mu_\lambda \sqrt{2} f_{Vn} = \langle 0 | \overline{q} \gamma^\mu q | \phi_{n,\lambda} \rangle$ for the plus component of the em current to minimize the contribution from possible zero-modes. The valence wave function, Eq. (5), is introduced in the formula for $f_{V,n}$ after integrating over $k^-$ in the momentum

loop. Another ingredient of our model is the probability of the VM valence component, roughly estimated by $\sim 1/\sqrt{2n+3/2}$ [4].

## 3 Results and Conclusion

The parameters in the form factor calculation are: i) the oscillator strength ($\omega = 1.39\ GeV^2$) and the constituent quark mass ($m = .265\ GeV$), constrained by the meson spectrum [3], and ii) a single vector meson width (equal to 0.15 $GeV$) for the vector mesons with $M_n \geq 2.150\ GeV$, while the experimental widths are used in the other cases.

The results of our model for the pion form factor show an overall agreement both with the TL and SL data [4], from -10 $(GeV/c)^2$ to 10 $(GeV/c)^2$. In Fig. 1, a very detailed comparison of our model in the SL region is presented. In particular, the ratio between the pion form factor, $F_\pi(q^2)$ and a monopole, i.e. $R_\pi(q^2) = F_\pi(q^2)/(1 - \frac{q^2}{m_\rho^2})^{-1}$, is adopted in order to avoid a log plot and therefore appreciate the virtue and drawbacks of our approach. In Fig. 1, on the left, $R_\pi(q^2)$ is shown over the whole range of the experimental data ( as collected in Ref. [8]). The ratio tends asymptotically to a constant in agreement with the expected behavior in QCD (see Ref.[2]); on the right, $R_\pi(q^2)$ is compared with the low momentum transfer data, in order to investigate the charge radius.

In summary, the model reproduces satisfactory the experimental data in the space-like region, while the agreement with the data in the time-like region is only found at the qualitative level. The next step is to improve the quality of the description in the time-like region, introducing isospin mixing and possible coupling of the photon to ${}^3D_1$ mesons.

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